Robustness Characteristics of Optimum Structural/Control Design

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An optimization program has been developed to design a minimum-weight flexible structure with constraints imposed on the closed-loop damping parameters, eigenvalue distribution, and the robustness bounds of the control system to retain stability under structured uncertainties. The formulation of approximate analytical expressions for the derivatives of the robustness measure is presented. The optimization problem was solved by using a nonlinear mathematical optimization technique. The numerical results are presented for the Draper 1 tetrahedral truss structure to satisfy different design requirements. These results indicate that the constraints on the damping parameters have to be imposed in addition to the robustness measure in order to improve the dynamic response.

Introduction

THE traditional approach to the design of a structure and control system has been sequential. The structure is first designed to satisfy the constraints on the allowable stresses, displacements, frequencies, etc., and then the control system is designed on this fixed structure. The control design has to satisfy its design requirements such as performance, control effort, robustness, stability, etc. This sequential approach generally does not lead to an optimum performance of the structure and control system for a specific mission. The optimum structural design and optimum control design have developed into mature fields. However, because of the stringent requirements in the design of future aerospace structures, there is currently considerable interest in developing algorithms for the simultaneous design of the structure and control system.

The mathematical model representing a physical system inevitably contains inaccuracies. This would compromise the performance of the control system, making the system unstable if one does not allow for these modeling errors. The modeling uncertainties are generally divided into two broad categories. The first is the parametric or structured uncertainties that result due to the variations in the real and analytical frequencies of the structure. The second is due to the unmodeled higher frequencies. The robust-control system is designed to maintain closed-loop stability and performance in the presence of unknown or known structured and unstructured uncertainties. The order of the finite element model of a space structure is generally too high and this necessitates the use of a reduced-order model constructed by truncating the higher vibration modes to design a controller. This reduction in the dimensionality of the controller sometimes leads to the spillover effect when it is used on a higher order structural model. There have been various approaches proposed by different investigators in order to reduce or prevent the spillover effect. The objective of this paper was to develop an optimization algorithm to design a structure and control system by taking into consideration a robustness parameter that defines the structural uncertainties in a closed-loop system. The linear quadratic regulator (LQR) with a full-order controller is used in the illustrative examples in order to avoid the effect of unmodeled dynamics on the optimum design of the structure and control system. The problem of optimum design of the structure and control system taking into consideration the unmodeled dynamics would be addressed in the future.

In recent years, the study of structure and control interaction has received much attention.[1,2] A number of investigators have considered an integrated approach to the optimum design of the structure and control systems for space structures.[3-13] Most of these investigations did not consider the robustness characteristics of the control design. Lim and Junkins[14] considered structure/control design by using the robustness parameter defined by Patel and Toda[15] and minimized the eigenvalue sensitivity for improving the stability robustness. They concluded that the stability robustness measure produces more robust design than minimizing the eigenvalue sensitivities. The robustness parameter defined by Patel and Toda[15] requires the solution of the Lyapunov equations. Rew et al.[16] presented a pole-placement technique for obtaining a robust eigenstructure using state energy, control energy, and stability robustness measure as the objective functions. Grandhi et al.[17] considered integrated structural/control design taking into consideration the robustness parameter defined by Qiu and Davison.[18] They used a finite difference approach to obtain the gradients of the constraints, which was not computationally efficient.

This paper presents the algorithm to design a minimum-weight structure with constraints on the closed-loop frequency distribution, robustness parameters, and damping parameters using analytical expressions for the gradients. The primary design variables are the cross-sectional areas of the members. The robustness parameter for structured uncertainty defined by Juang et al.[19] is used. This parameter is similar to the one used in Ref. 17. It is a function of the spectral radius of the matrix rather than depending on the solution of the Lyapunov equations. Analytical expressions for the sensitivities of the robustness parameter measure are derived with some approximations. For illustration of the algorithm, the Draper 1 tetrahedral truss structure was used. The structure was designed with constraints with increasing requirements on the robustness measure. The algorithm was found to give optimum solutions satisfying the constraints accurately. The transient response for a specified initial disturbance is studied for the optimum designs.

Problem Formulation

The equations describing the dynamic behavior of a structure are as follows:

$$\dot{x} = Ax + Bf \tag{1}$$

Received July 25, 1990; presented as Paper 90-3358 at the AIAA Guidance, Navigation, and Control Conference, Portland, OR, Aug. 20-22, 1990; revision received Dec. 4, 1990; accepted for publication Feb. 20, 1991.

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$$y = Cx \tag{2}$$

where x is the state vector, f the control vector, and y the output vector. In Eq. (1), A and B are the plant and input matrices given by

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -\omega^2 & -2\zeta\omega \end{array}\right] \tag{3}$$

$$B = \left[\begin{array}{c} 0 \\ \hline \phi^T D \end{array}\right] \tag{4}$$

where I is the identity matrix, ω^2 the diagonal matrix of squares of structural frequencies, ζ the modal damping factor, and ϕ the modal matrix. In Eq. (2), $C = B^T$ if the sensors and actuators are collocated. The optimal control f is selected to minimize the following performance index integral:

$$J = \int_0^\infty (x^T Q x + f^T R f)\, \mathrm{d}t \tag{5}$$

where Q and R are a positive state weighting matrix and a positive definite control weighting matrix, respectively. The optimum feedback control law is given by

$$f = -Gx \tag{6}$$

where G is the optimum gain matrix given by

$$G = R^{-1} B^T P \tag{7}$$

In Eq. (7), P is the Riccati matrix obtained by the solution of the algebraic Riccati equation:

$$0 = Q + PA + A^T P - PBR^{-1}B^T P \tag{8}$$

Using Eqs. (1) and (6), the optimum closed-loop system can be written as

$$\dot{x} = \bar{A} x \tag{9}$$

where

$$\bar{A} = A - BG \tag{10}$$

or

$$\bar{A} = A - XP \tag{11}$$

where

$$X = BR^{-1}B^T \tag{12}$$

The complex eigenvalues of $\bar{A}$ can be written as

$$\lambda_i \tilde{\sigma}_i \pm j\tilde{\omega}_i \tag{13}$$

The damping factor of the closed-loop system is given by

$$\xi_i = -\frac{\tilde{\sigma}_i}{(\tilde{\sigma}_i^2 + \tilde{\omega}_i^2)^{1/2}} \tag{14}$$

The perturbed closed-loop system can be written as

$$\dot{x} = (\bar{A} + \epsilon)x \tag{15}$$

where $\bar{A}$ is a stable matrix and ϵ is a perturbation matrix. The perturbed system would be stable if the elements ϵ_{ij} satisfy the condition[19]

$$\epsilon_{ij} < \frac{1}{\sup\limits_{p \geq 0} \rho[\,|(jpI - \bar{A})^{-1}|\,U_e]} \cdot U_{e_{ij}} = \frac{1}{\rho_s} \cdot U_{e_{ij}} \tag{16}$$

where $|(\,)|$ denotes the absolute matrix, $\rho[\,]$ denotes the spectral radius of the matrix $[\,]$, $U_{e_{ij}}$ are the elements of the perturbation identification matrix U_e, and sup represents the supremum of the matrix over a range of p. The elements of the perturbation identification matrix U_e would vary from 0 to 1. If any element of $\bar{A}$ is constant or zero, then the associated element of U_e would be equal to zero, indicating that there is no perturbation. For structural problems, the elements of the top half of the $\bar{A}$ matrix are either equal to zero or unity. Thus, the elements of matrix U_e associated with these elements would be equal to zero. The remaining elements would lie between zero and unity. Selection of elements equal to unity gives a conservative measure of the robustness. Yedavalli et al.[20] have suggested using the absolute normalized values of the elements of the matrix $\bar{A}$ so that

$$U_{e_{ij}} = \frac{|\bar{A}_{ij}|}{|\bar{A}_{ij}|_{\max}} \tag{17}$$

where $|\bar{A}_{ij}|_{\max}$ is the element of the matrix $\bar{A}$ with the maximum absolute value.

In order to find the robustness parameter, it is necessary to calculate ρ_s in Eq. (16). This can be done by finding the eigenvalues and taking their modulus. The spectral radius is the maximum of these values for a selected value of the operating frequency p. The maximum spectral radius value among all values of p defines ρ_s. The critical operating frequency associated with ρ_s is denoted by p_s. In Ref. 17, it is shown that, for structural problems, the peaks in the spectral radius plot occur only at the p equal to the modulus of the closed-loop frequencies of the matrix $\bar{A}$. This assumption reduces the computational effort needed to determine ρ_s and also its sensitivity with respect to the design variables; it makes the optimization of the structural controller problem amenable to a solution.

Sensitivity Calculations

The sensitivity of the closed-loop eigenvalues with respect to the structural design variable A_l is given by

$$\lambda_{i,l} = \beta_i^T \bar{A}_{,l} \alpha_i \tag{18}$$

where β_i and α_i are the left and right eigenvectors of $\bar{A}$. Using Eq. (11), the partial derivative of $\bar{A}$ with respect to A_l can be written as

$$\bar{A}_{,l} = A_{,l} - X_{,l}P - XP_{,l} \tag{19}$$

Matrices A and X are functions of the structural frequency ω_j and the modal matrix $[\phi]$ for which the sensitivities are given by

$$\frac{\partial \omega_j^2}{\partial A_l} = \frac{1}{A_l} \{\phi_j\}_{,l}^T [k_l - \omega_j^2 m_l] \{\phi_j\}_{,l} \tag{20}$$

$$\{\phi_j\}_{,l} = \sum_{i=1}^{n} \alpha_{ijl} \{\phi_i\} \tag{21}$$

where

$$\alpha_{ijl} = \frac{1}{A_l(\omega_i^2 - \omega_j^2)} \{\phi_j\}_{,l}^T [k_l - \omega_i^2 m_l] \{\phi_i\}_{,l}, \qquad i \neq j \tag{22}$$

$$\alpha_{iil} = -\frac{1}{A_l} \{\phi_i\}_{,l}^T m_l \{\phi_i\}_{,l} \tag{23}$$

In Eq. (20), k_l and m_l represent the element stiffness and mass matrix of element A_l, respectively. Differentiating Eq. (8) with respect to the design variable and using Eq. (19) gives

$$\bar{A}^T P_{,l} + P_{,l} \bar{A} = \tilde{B} \tag{24}$$

where

$$\tilde{B} = -A_{,l}^T P - PA_{,l} + PX_{,l}P \tag{25}$$

The solution of Lyapunov Eq. (24) gives the sensitivity of the Riccati matrix $\boldsymbol{P}$. Substituting this in Eq. (19) gives the sensitivity of the closed-loop matrix $\bar{\boldsymbol{A}}$.

The spectral radius ρ_s is not differentiable with respect to the operating frequency p at the actual maximum. The spectral radius and the elements of the perturbation identification matrix are assumed to be continuous functions of the design variables A_l for the purpose of evaluating their sensitivities. It is also assumed that the critical operating frequency p_s is known and the sensitivities are calculated for that specified value of p_s with the assumption that it is invariant for small changes in the design variables. The use of the analytical expressions for sensitivities derived based on these assumptions gave optimum designs satisfying the constraints to a high degree of accuracy. If the calculation of sensitivities would have been erroneous, the optimization procedure would have never converged.

The largest eigenvalue of $|(jp_s\boldsymbol{I} - \bar{\boldsymbol{A}})^{-1}| \cdot \boldsymbol{U}_e$ for the specified critical operating frequency p_s can be written as

$$\lambda_\rho = \lambda_\rho^r + j\lambda_\rho^i \tag{26}$$

where λ_ρ^r and λ_ρ^i are the real and imaginary parts of λ_ρ. Then the spectral radius ρ_s can be written as

$$\rho_s = [(\lambda_\rho^r)^2 + (\lambda_\rho^i)^2]^{1/2} \tag{27}$$

The sensitivity of ρ_s with respect to the lth design variable can be written as

$$\rho_{s,l} = \frac{\lambda_\rho^r\lambda_{\rho,l}^r + \lambda_\rho^i\lambda_{\rho,l}^i}{\rho_s} \tag{28}$$

The sensitivity of the eigenvalues λ_ρ can be written as

$$\lambda_{\rho,l} = \boldsymbol{\beta}_\rho \bar{\bar{\boldsymbol{A}}}_{\rho,l}\boldsymbol{\alpha}_\rho \tag{29}$$

where

$$\bar{\bar{\boldsymbol{A}}}_\rho = \bar{\boldsymbol{A}}_\rho \cdot \boldsymbol{U}_e \tag{30}$$

where

$$\bar{\boldsymbol{A}}_\rho = |(jp_s\boldsymbol{I} - \bar{\boldsymbol{A}})^{-1}| \tag{31}$$

In Eq. (29), $\boldsymbol{\beta}_\rho$ and $\boldsymbol{\alpha}_\rho$ are the left and right eigenvectors of $\bar{\bar{\boldsymbol{A}}}_\rho$. Differentiating Eq. (30) with respect to the design variable A_l gives

$$\bar{\bar{\boldsymbol{A}}}_{\rho,l} = \bar{\boldsymbol{A}}_{\rho,l} \cdot \boldsymbol{U}_e + \bar{\boldsymbol{A}}_\rho \cdot \boldsymbol{U}_{e,l} \tag{32}$$

The elements of matrix $\bar{\boldsymbol{A}}_\rho$ can be written as

$$\bar{A}_{\rho_{ij}} = [(a_{\rho_{ij}}^r)^2 + (a_{\rho_{ij}}^i)^2]^{1/2} \tag{33}$$

where $a_{\rho_{ij}}^r + ja_{\rho_{ij}}^i$ are the elements of the complex matrix $\boldsymbol{A}_\rho = (jp_s\boldsymbol{I} - \bar{\boldsymbol{A}})^{-1}$. The sensitivity of $\bar{A}_{\rho_{ij}}$ can be written as

$$\bar{A}_{\rho_{ij,l}} = \frac{a_{\rho_{ij}}^r a_{\rho_{ij,l}}^r + a_{\rho_{ij}}^i a_{\rho_{ij,l}}^i}{\bar{A}_{\rho_{ij}}} \tag{34}$$

The sensitivity of $\boldsymbol{A}_\rho$ is given by

$$\boldsymbol{A}_{\rho,l} = -\boldsymbol{A}_\rho \boldsymbol{A}_{\rho,l}^{-1}\boldsymbol{A}_\rho \tag{35}$$

where

$$\boldsymbol{A}_{\rho,l}^{-1} = [jp_s\boldsymbol{I} - \bar{\boldsymbol{A}}]_{,l} \tag{36}$$

For a specified value of p_s,

$$\boldsymbol{A}_{\rho,l}^{-1} = -\bar{\boldsymbol{A}}_{,l} \tag{37}$$

The sensitivity of $\bar{\boldsymbol{A}}_{,l}$ is given in Eq. (19).

In Eq. (16), if the elements of the perturbation identification matrix $\boldsymbol{U}_e$ are constant, then $\boldsymbol{U}_{e,l}$ would be zero in Eq. (30). In the case where the elements are defined as given in Eq. (17), it is required to calculate the sensitivities of the element of $\boldsymbol{U}_e$. Taking into consideration the sign of the elements $\bar{A}ij$ and $\bar{A}_{ij,l}$, the sensitivity of the elements of $\boldsymbol{U}_e$ can be written:

$$U_{e_{ij,l}} = \frac{|\bar{A}_{ij}|_{,l}|\bar{A}_{ij}|_{\max} - |\bar{A}_{ij}||\bar{A}_{ij}|_{\max,l}}{(|\bar{A}_{ij}|_{\max})^2} \tag{38}$$

Optimization

The optimization problem for the integrated design of the structure and control system can be stated as the following: Determine n_1 design variables A_i that would minimize the structural weight,

$$W = \sum \rho_i A_i \ell_i, \qquad i = 1, \ldots, n_1 \tag{39}$$

and satisfy the constraints

$$g_j(\tilde{\omega}_i) = \tilde{\omega}_i - \bar{\tilde{\omega}}_i \geq 0, \qquad j = 1, \ldots, m_1 \tag{40}$$

$$g_j(\xi_i) = \xi_i - \bar{\xi}_i \geq 0, \qquad j = 1, \ldots, m_2 \tag{41}$$

$$g(\rho_s) = \bar{\rho}_s - \rho_s \geq 0 \tag{42}$$

$$g_j(A_i) = A_i - \bar{A}_i \geq 0, \qquad j = 1, \ldots, n_1 \tag{43}$$

in Eq. (39), ρ_i is the density and ℓ_i is the length of the ith element. In Eqs. (40–42), $g_j(\tilde{\omega}_i)$, $g_j(\xi_i)$, and $g(\rho_s)$ are the constraints on the imaginary part of the closed-loop eigenvalues, on the damping parameters, and on the spectral radius, respectively, and $\bar{\tilde{\omega}}_i$, $\bar{\xi}_i$, and $\bar{\rho}_s$ are the limiting values of the eigenvalues, damping parameters, and the spectral radius, respectively. For convenience, the constraint is imposed on the spectral radius ρ_s instead of the stability bounds ϵ_{ij}. A decrease in ρ_s would increase the robustness requirement of the control system. In Eq. (43), $\bar{A}_i$ is the minimum allowable cross-sectional area of the element. The optimization problem was solved by using the NEWSUMT-A[21] optimization program, which is based on the extended interior penalty function method with Newton's method of unconstrained minimization.

In the NEWSUMT-A package, the sensitivities of the objective functions can be calculated by using a finite difference scheme or the user can provide them. The finite difference approach is computationally inefficient when the number of design variables is large. This option is generally suitable when analytical expressions cannot be derived because of the nature of the problem to be solved. For our problem, the sensitivities of the objective function can be obtained by differentiating Eq. (39) with respect to the design variable A_l. The sensitivities of the constraints can be obtained by using the relations derived in the last section.

The major steps involved in structure/control optimization for the problem solved in this paper are the following:

1) For the specified values of the cross-sectional areas of the members, structural frequencies ω_j and the vibration modes ϕ and their sensitivities with respect to the cross-sectional area of the members were calculated.

2) The plant matrix $\boldsymbol{A}$ and the input matrices $\boldsymbol{B}$, in the state-space input equation, and the output matrix $\boldsymbol{C}$, in the state output equation, were calculated.

3) The linear optimum regulator control problem was solved and the closed-loop eigenvalues, eigenvectors, and damping factors were determined.

4) The spectral radius for the operating frequencies p that are equal to the modulus of the eigenvalues of the closed-loop matrix $\bar{\boldsymbol{A}}$ were evaluated. The maximum value among all these values is equal to ρ_s and the associated operating frequency is equal to p_s.

5) The sensitivities of the closed-loop eigenvalues, damping parameters, and the ρ_s were calculated as needed.

6) The design variables were modified by using a suitable algorithm.

7) With the new values of the design variables, steps 1-6 were repeated until the optimum solution satisfying all of the specified constraints was obtained. The repetition of these steps was required because of the nonlinear nature of the problem.

Example: Draper 1 Tetrahedral Truss

The finite element model of the tetrahedral truss is shown in Fig. 1. This model represents a typical flexible structure pointing in a specific direction. The structure has no rigid degrees of freedom. It has 10 nodes and 12 elements, which are assumed to be truss elements carrying no bending moments. The coordinates of the node points are given in Table 1. The dimensions and the elastic properties for this structure are specified in consistent nondimensional units. Young's modulus is assumed equal to 1.0 and the density of the structural material is 0.001. Four nonstructural masses of 2.0 units are located at nodes 1-4. The apex (node 1) represents the antenna feed and its motion in the X-Y plane defines the line-of-sight (LOS) errors. If the motion of node 1 in the X and Y directions is denoted by LOS_X and LOS_Y, respectively, then $\mathrm{LOS} = [(\mathrm{LOS}_X)^2 + (\mathrm{LOS}_Y)^2]^{1/2}$. Since the structure has 12 degrees of freedom, the state-space model would be of the 24th order. The structural damping parameter ζ in Eq. (3) for the illustrative problem is assumed to be zero for all of the modes. The weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ for the state variables and control variables are assumed to be identity matrices.

The cross-sectional areas of the initial design for optimization are assumed equal to those assigned by the Charles Stark Draper Laboratory. During the discussion of the results, this design is referred to as the nominal design (design 1). This structure was first analyzed and the optimal control was designed. This design weighs 43.69 units. The complex closed-loop eigenvalues for the lowest two modes for the design are

$$-0.07341 \pm j1.3414 \tag{44a}$$

$$-0.10895 \pm j1.6630 \tag{44b}$$

The closed-loop damping associated with these modes equals 0.05464 and 0.06537. The spectral radius ρ_s for the nominal

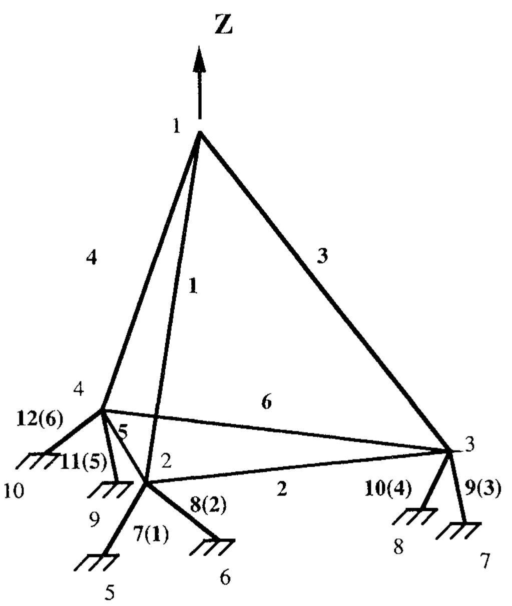

Fig. 1 Tetrahedral truss structural model (actuator numbers are in parentheses).

Table 1 Node point coordinates

Node	X	Y	Z
1	0.0	0.0	10.165
2	− 5.0	− 2.887	2.0
3	5.0	− 2.887	2.0
4	0.0	5.7735	0.0
5	− 6.0	− 1.1547	0.0
6	− 4.0	− 4.6188	0.0
7	4.0	− 4.6188	0.0
8	6.0	− 1.1547	0.0
9	− 2.0	5.7735	0.0
10	2.0	5.7735	0.0

design is equal to 0.4771, and the critical operating frequency p_s is equal to 12.90. The maximum absolute value of the element $|\bar{A}_{ij}|$ is 166.81 and its location is $i = 24$ and $j = 12$. This is the same location as $(\omega_{12})^2$ in the plant matrix A in Eq. (3) where ω_{12} is the maximum structural frequency. Hence, the element $U_e(24,12)$ is equal to unity and all other elements would be smaller than 1. When the constraint is specified on the spectral radius ρ_s, it is primarily associated with this element, and the robustness bound ϵ_{ij} for this element would be $1/\rho_s$. For other elements in the matrix $\bar{A}$, the robustness bound ϵ_{ij} would be smaller than $1/\rho_s$ since $U_{e_{ij}}$ associated with these elements are smaller than unity.

The constraints on the optimum design were imposed initially only on $\tilde{\omega}_1$, $\tilde{\omega}_2$, and ρ_s as follows:

$$\tilde{\omega}_1 \geq 1.34 \tag{45a}$$

$$\tilde{\omega}_2 \geq 1.6 \tag{45b}$$

$$\rho_s \leq \bar{\rho}_s = \alpha\rho_s(\text{initial}) \tag{45c}$$

ρ_s (initial) denotes the spectral radius associated with the initial design (design I), which is equal to 0.4771. In order to specify different constraints on the robustness, α in Eq. (45c) was set equal to 0.5, 0.4, 0.3, 0.2, and 0.1, respectively. These designs would be designated as designs II–VI. Decreasing the value of α imposes higher robustness requirements. Design VII was obtained by imposing the following constraints:

$$\tilde{\omega}_1 \geq 1.34 \tag{46a}$$

$$\tilde{\omega}_2 \geq 1.6 \tag{46b}$$

$$\rho_s \leq 0.5\rho_s(\text{initial}) = 0.2386 \tag{46c}$$

$$\xi_1 \geq 0.1093 \tag{46d}$$

$$\xi_2 \geq 0.1307 \tag{46e}$$

The last two constraints were imposed to improve the closed-loop damping associated with first two closed-loop frequencies. These values are twice those for design I. As we will see, the designs with constraints only on the frequencies $\tilde{\omega}_1$, $\tilde{\omega}_2$, and ρ_s have generally smaller closed-loop damping than the nominal design (design I). The iteration history for the six optimum designs is given in Table 2. The constraints specified in Eq. (45) or (46) were satisfied as nearly equality constraints with an error of the order of 10^{-4}–10^{-5}. The bottom two rows of Table 2 give the spectral radius ρ_s and associated critical frequency p_s for all of the optimum designs. The number of iterations needed to obtain an optimum design increased with a decreasing value of α or decreasing the required spectral radius parameter ρ_s. The weight of the optimum designs increases with a reduction in ρ_s. The critical operating frequency p_s was not the same for all optimum designs. This may be due to the band of structural frequencies not being the same for all of the designs. Comparing designs II and VII, where the first three constraints are identical, and imposing constraints to improve the closed-loop damping increases the weight by nearly 83%.

Table 3 gives the cross-sectional area of the elements of the structure for all of the optimum designs. Comparing the relative cross-sectional areas for each design, it is observed that the improvement in the robustness is achieved by redistribution of the areas among different elements and not by proportional scaling of the cross-sectional areas with increase in the weight of the structure. The cross-sectional area of element 7 or 8 is larger than those of other elements for designs II-VI. For design VI where $\alpha = 0.1$, the cross-sectional area of element 8 is 5161.23, which is substantially higher than the other elements. This design had the maximum robustness requirement. The optimization procedure stiffened this element in order to increase the frequency associated with the corresponding vibration mode.

The distribution of the square of the structural frequencies given in Table 4 indicates that ω_1^2 and ω_2^2 are nearly the same for all the designs. This is due to the constraints on $\tilde{\omega}_1$ and $\tilde{\omega}_2$ in Eqs. (45) and (46). For the initial design, the square of the structural frequencies lies between 1.8 and 166.5. For optimum designs II-VI, the first 11 frequencies lie within 1.8 and about 48.95. The ω^2 associated with last mode increases with a reduction in ρ_s or an increase in the robustness requirements of the design. For example, for $\rho_s = 0.5$, the maximum ω^2 was 76.97, whereas for $\rho_s = 0.1$, this value increased to 918.10. In design VII, where the constraints on the damping parameters are imposed, the ω^2 lies between 1.80 and 109.70.

The closed-loop damping parameters associated with all of the modes are given in Table 5. Comparing designs II-VI, it is observed that there is no specific trend in the damping parameters with an increase in the robustness requirements.

Table 2 Iteration history of structural weight

Iteration number	Design II	Design III	Design IV	Design V	Design VI	Design VII
1	43.69	43.69	43.69	43.69	43.69	43.69
2	17.70	18.83	19.98	22.32	24.61	16.94
3	14.98	15.23	15.54	17.88	19.07	16.24
4	14.64	14.83	15.28	17.67	19.28	19.55
5	14.64	14.82	15.25	17.67	24.44	23.21
6				17.35	26.43	25.09
7				18.11	27.35	26.65
9				18.07	27.81	26.82
10				18.06	27.88	26.89
11					27.91	26.91
α	0.5	0.4	0.3	0.2	0.1	0.5
ρ_S	0.2386	0.1908	0.1431	0.09542	0.04771	0.2386
p_S	2.77	1.60	1.60	15.48	30.30	9.19

Table 3 Cross-sectional areas of the members

Iteration number	Design I	Design II	Design III	Design IV	Design V	Design VI	Design VII
1 (1-2)	1000.0	205.51	214.94	219.53	344.21	210.27	446.97
2 (2-3)	1000.0	216.55	207.68	199.35	134.40	199.39	531.56
3 (1-3)	100.0	128.68	130.20	129.46	161.73	130.79	190.25
4 (1-4)	100.0	204.51	201.06	199.95	158.15	193.04	448.44
5 (2-4)	1000.0	136.99	139.06	139.17	135.72	139.75	178.49
6 (3-4)	1000.0	223.88	233.54	235.54	250.16	230.37	492.40
7 (2-5)	100.0	130.63	127.00	129.10	1310.80	127.72	256.33
8 (2-6)	100.0	397.15	467.99	631.31	180.71	5161.23	511.10
9 (3-7)	100.0	182.05	161.53	154.77	143.31	173.50	39.99
10 (3-8)	100.0	184.33	176.60	173.90	206.19	178.91	40.63
11 (4-9)	100.0	129.25	123.45	128.24	150.03	120.95	59.54
12 (4-10)	100.0	206.99	202.44	206.03	206.52	204.79	517.21
Weight	43.70	14.64	14.82	15.25	18.06	27.91	26.91
α	1.0	0.5	0.4	0.3	0.2	0.1	0.5

Table 4 Structural frequencies

Mode	Design I	Design II	Design III	Design IV	Design V	Design VI	Design VII
1	1.80	1.80	1.80	1.80	1.80	1.80	1.80
2	2.77	2.56	2.56	2.56	2.56	2.56	2.56
3	8.35	7.68	7.68	7.65	8.16	7.79	5.20
4	8.74	9.38	9.20	9.22	8.76	9.04	9.41
5	11.55	13.13	13.26	13.47	16.51	13.43	16.36
6	17.68	25.67	25.27	25.52	26.06	25.02	19.39
7	21.73	28.39	27.34	27.53	29.26	27.66	28.33
8	22.61	35.76	33.80	33.15	39.09	34.66	39.72
9	72.92	40.66	39.27	38.85	41.50	39.34	60.38
10	85.57	41.68	40.94	41.01	44.34	41.20	84.54
11	105.8	45.43	44.63	45.05	48.95	44.61	103.30
12	166.5	76.97	89.47	118.10	239.9	918.10	109.70
α	1.0	0.5	0.4	0.3	0.2	0.1	0.5

Table 5 Modal damping parameters of the closed-loop system

Mode	Design I	Design II	Design III	Design IV	Design V	Design VI	Design VII
1	0.0546	0.0386	0.0404	0.0397	0.0329	0.0403	0.1093
2	0.0653	0.0295	0.0311	0.0311	0.0301	0.0295	0.1307
3	0.0737	0.0297	0.0321	0.0323	0.0282	0.0315	0.1164
4	0.0801	0.0487	0.0509	0.0496	0.0382	0.0502	0.0747
5	0.0839	0.0337	0.0352	0.0345	0.0353	0.0336	0.0747
6	0.0864	0.0515	0.0529	0.0510	0.0471	0.0538	0.0587
7	0.0760	0.0649	0.0659	0.0666	0.0658	0.0641	0.0357
8	0.0723	0.0543	0.0582	0.0596	0.0507	0.0555	0.0383
9	0.0341	0.0518	0.0538	0.0539	0.0527	0.0536	0.0344
10	0.0291	0.0436	0.0395	0.0386	0.0504	0.0408	0.0209
11	0.0207	0.0479	0.0476	0.0482	0.0403	0.0491	0.0347
12	0.0064	0.0402	0.0373	0.0325	0.0228	0.0116	0.0325
α	1.0	0.5	0.4	0.3	0.2	0.1	0.5

For design VII, the closed-loop damping parameters associated with the first two frequencies are twice those of design I. This is due to the constraints imposed on this design. Designs VI and VII weigh nearly the same; however, the damping associated with the first three closed-loop frequencies of design VII were substantially greater than those of design VI.

In Eq. (45c) or (46c) the constraint was specified in terms of ρ_s. If this condition is converted into the element of the perturbation matrix, then $\epsilon_{24,12}(=1/\rho_s)$ would be equal to 2.096, 4.19, 5.24, 6.98, 10.48, 20.96, and 4.19 for designs I–VII, respectively. The percentage allowable perturbations in the element $\bar{A}$**(24, 12)** would then be equal to 1.26, 5.45, 5.80, 5.90, 4.38, 2.28, and 3.82 for designs I–VII, respectively. The percentage allowable perturbations were not proportional to the values of $\epsilon_{24,12}$ because the magnitude of element $\bar{A}$**(24, 12)** was not the same for all of the designs. It may be noted that these percentages of the allowable perturbations would be the same for all of the elements of the closed-loop $\bar{A}$ in a specific design. This is due to the nature of the perturbation identification matrix U_e.

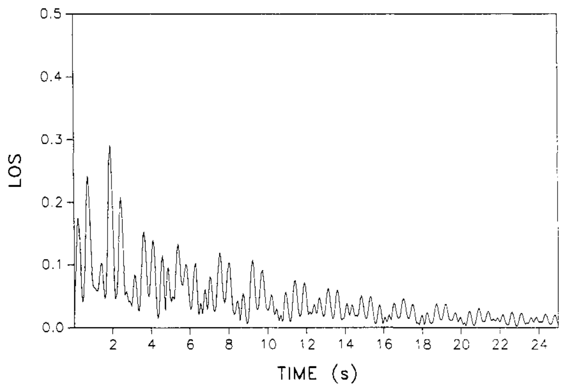

Fig. 2 Transient response to design I.

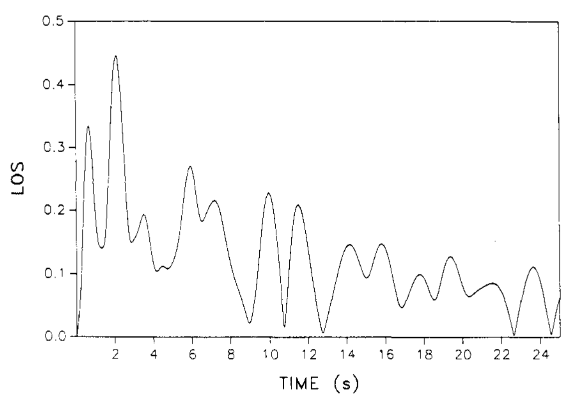

Fig. 3 Transient response to design II.

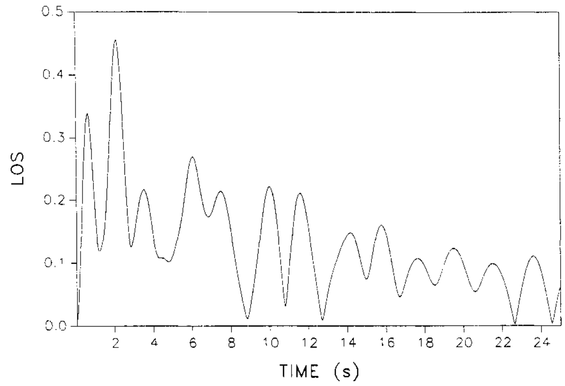

Fig. 4 Transient response to design VI.

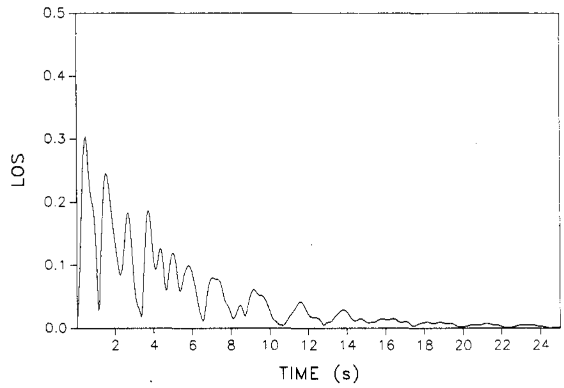

Fig. 5 Transient response to design VII.

In order to study the dynamic behavior of the optimum designs, a unit displacement was imposed at time $t=0$ at node 2 in the X direction. The transient response is simulated for designs I, II, VI, and VII for the period $y=25$ s at time interval $t=0.05$. The magnitude of the LOS is calculated at each time step. The time histories for the four designs are given in Figs. 2–5. Comparing the four designs shows that the dynamic response is better for design VII, which is designed to improve robustness as well as closed-loop damping. Designs II and VII have the same robustness requirement. However, the performance of design VII is better than that of design II. Designs VI and VII weigh nearly the same. Design VI is more robust than design VII. However, the dynamic response of design VII is better than design VI. The magnitudes of the performance index in Eq. (5) for designs I, II, VI, and VII were 764.4, 120.8, 412.9, and 239.1, respectively.

Concluding Remarks

The primary objective of this paper is to formulate an algorithm to design minimum-weight structures with constraints on closed-loop eigenvalues and damping and to improve robustness stability under structured uncertainties. The robustness characteristic of the system is defined by the maximum spectral radius of the matrix. The analytical expressions are formulated for the sensitivities of the robustness measure with some assumptions regarding the continuity of the function. Using these analytical sensitivities, the optimization algorithm converged to the optimum solution satisfying all constraints to a high degree of accuracy.

Numerical results are presented for the tetrahedral truss structure, which demonstrated the capability of the algorithm and brought about the salient feature of the optimum design. Only improving the robustness characteristic of the system leads to an unacceptable structural design with a very large variation in the cross-sectional areas of the elements. The closed-loop damping of the system does not substantially change with the increase in the robustness. Imposing the constraint on the damping parameter in addition to the robustness parameter gives a structure with better dynamic response. This indicates that constraining the robustness parameter without improving the damping parameters is not adequate to get a good design.

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